BayesTraits V2

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Major Changes from V1

A range of new models:

Multiple regression

Variable rates

Independent Contrast

Independent Contrast: Correlations

Independent Contrast: Regression

Covarion models

Higher precision method for large trees

Rate deviation and data deviation parameters are automatically tuned

Hard polytomys are now supported

Improved handling of model files

Estimation of internal and tips for continuous and discrete data

Numerous bug fixes and improvements

Disclaimer

Software development is a notoriously error prone activity. While efforts are made to make the programme as accurate as possible, due to the size and complexity of *BayesTraits* it will contain bugs, care must be taken when using the software. If you notice any unusual behaviour including crashes or inconsistent results please contact the authors with the data, trees and commands used.

Introduction

BayesTraits is a computer package for performing analyses of trait evolution among groups of species for which a phylogeny or sample of phylogenies is available, these can be created using BayesPhylogenies. It can be applied to the analysis of traits that adopt a finite number of discrete states, or to the analysis of continuously varying traits. The methods can be used to take into account uncertainty about the model of evolution and the underlying phylogeny. Evolutionary hypotheses can be tested including

Finding rates of evolution

Establishing correlations between traits

Calculating ancestral state values

Building regression models

Predicting unknown values

Testing for modes of evolution

Accelerated / decelerated rates of evolution through time

Magnitude of phylogenetic signal

Variable rate of evolution through time and within the tree

Ornstein-Uhlenbeck processes

If trait change is concentrated at speciation events

Test for covarion evolution

Methods and Approach

BayesTraits uses Markov chain Monte Carlo (MCMC) methods to derive posterior distributions and maximum likelihood (ML) methods to derive point estimates of, log-likelihoods, the parameters of statistical models, and the values of traits at ancestral nodes of phylogenies. The user can select either MCMC or reversible-jump MCMC. In the latter case the Markov chain searches the posterior distribution of different models of evolution as well as the posterior distributions of the parameters of these models (see below).

BayesTraits can be used with a single phylogenetic tree in which case only uncertainty about model parameters is explored, or, it can be applied to suitable samples of trees such that models are estimated and hypotheses are tested taking phylogenetic uncertainty into account.

BayesTraits is designed to be as flexible as possible, but users must treat this flexibility with care, as it allows models which may not have a valid interpretation, for example you care create complex models which cannot be estimated from the given data, such as estimating 100 parameters for a 30 taxa tree or build a regression model form unsuitable data.

BayesTraits methods

• <u>MultiState</u> is used to reconstruct how traits that adopt a finite number of discrete states evolve on phylogenetic trees. It is useful for reconstructing ancestral states and for testing models of trait evolution. It can be applied to traits that adopt two or more discrete states (Pagel, Meade et al. 2004)

• <u>Discrete</u> is used to analyse correlated evolution between pairs of discrete binary traits. Most commonly the two binary states refer to the presence or absence of some feature, but could also include "low" and "high", or any two distinct features. Its uses might include tests of correlation among behavioural, morphological, genetic or cultural characters (Pagel 1994, Pagel and Meade 2006)

• <u>Continuous</u> is for the analysis of the evolution of continuously varying traits using a GLS framework. It can be used to model the evolution of a single trait, to study correlations among pairs of traits, or to study the regression of one trait on two or more other traits (Pagel 1999).

• <u>Continuous regression</u> is used to build regression models and use these models to reconstruct unknown values (Organ, Shedlock et al. 2007).

• <u>Independent contrast</u> methods (Felsenstein 1973, Freckleton 2012) provides a very fast alternative to the GLS methods. They are useful for analysing large trees, but some model parameters cannot be estimated.

• <u>Variable rates</u> is used to detect variations in the rate of evolution threw the tree, accounting for changes in rate on a single lineage or for a group of taxa (Venditti, Meade et al. 2011).

This manual is designed to show how to use the programs that implement these models. Detailed information about the methods can be found in the papers listed at the end (some are available as pdfs on our website). Syntax and a description of all of the commands in BayesTraits is listed below.

Model	Data
MultiState	Multistate
Discrete: Independent	Two binary traits
Discrete: Dependent	Two binary traits
Continuous: Random Walk	Continuous traits
Continuous: Directional	Continuous traits
Continuous: Regression	Two or more continuous traits
Independent Contrasts	Continuous traits
Independent Contrasts: Correlation	Two or more continuous traits
Independent Contrasts: Regression	Two or more continuous traits
Discrete: Covarion	Two binary traits

Build information

BayesTraits comes in a number of different versions and a number of these versions can be combined. All versions of *BayesTraits* are for 64 bit operating systems, 32 bit versions are available on request. Different versions include,

Quad – a high precision version, for use with discrete and multistate models when using large trees, see Precision for more information.

OpenMP – a multicore version of BayesTraits for use with discrete and multistate model when using large trees, see OpenMP for more information.

OpenCL – a version of BayesTraits designed to use graphics cards to accelerate the computationally intensive sections of the program, see OpenCL for more information.

The different versions of *BayesTraits* may require external librarys to be installed on different operating systems, in most cases it's when using Linux versions.

Tree Format

BayesTraits requires trees to be in Nexus format, trees can include hard polytomies but must be correctly rooted and include branch lengths. Taxa names must not be included in the description of the tree but should be linked to a number in the translate section of the tree file, a number of example trees are included with the program.

Data Format

Data are read from a plain text file (ASCII), with one line for each species or taxon in the tree. The names must be spelled exactly as in the trees and must not have any spaces within them. They do not have to be in the same order. Following a species name, leave white space (tab or space) and enter the first column of data, repeat this for additional columns of data (see below). Data for MultiState analysis should take values such as "0", "1", "2" or "A", "B", "C" etc. Data for discrete must be exactly two columns of binary data and must take the values "0" or "1". Continuous data should be integers or floating points. If data are missing it should be represented using "-", for a trait in MultiState or Discrete the remaining data for the taxa is used, if data are missing for Continuous

the taxa are removed from the tree. Example data files for MultiState, Discrete and Continuous data are included with the program.

Example of MultiState data

Taxon01	А	А	С
Taxon02	В	В	С
Taxon03	Α	В	-
Taxon04	С	С	В
TaxonN	BC	Α	В

Taxon 3 has missing data for the third site. In Discrete and Multistate missing data are treated as if the trait could take any of the other states, with equal probability. Alternatively you can indicate uncertainty about a traits value. For example, the first trait for Taxon n is uncertain. The code BC signifies that it can be in states B or C (with equal probability) but not in state A.

Taxon01	0	0
Taxon02	0	-
Taxon03	1	0
Taxon04	0	1

1

Example of Discrete (binary) data

Example of Continuous data

TaxonN

Taxon01	10	9.0	
Taxon02	1.06	-	
Taxon03	5.3	2	
Taxon04	3	4	
TaxonN	1	1.1	

Running BayesTraits

BayesTraits is run from the command prompt (windows) or terminal (OS X and Linux), it is not run by double clicking on it. The program, tree file and data file should be place in the same directory / folder. Start the command prompt / terminal and change to the directory that the program, tree and data are in and type.

Windows

BayesTraits.exe TreeFile DataFile

Linux / OSX

./BayesTraits TreeFile DataFile

Where TreeFile is the name of the tree file and DataFile is the name of the data file.

Running BayesTraits with a command file

If you need to run an analysis multiple times or if it is complex it can be more convenient to place the commands into a command file, instead of typing them in each time. A command file is a plane ASCII text file, with the commands to run.

An example command file is included with the program, "ArtiodactylMLIn.txt". The file has three lines. The first line selects MultiState, the second is for ML analysis and the third is to run the program. To run *BayesTraits* using the Artiodacty tree, data and input file use the following command. The command and there order can be found by running the program normally and noting your inputs.

```
1
1
run
Windows
```

BayesTraits.exe Artiodactyl.trees Artiodactyl.txt < ArtiodactylMLIn.txt</pre>

Linux / OS X

./BayesTraits Artiodactyl.trees Artiodactyl.txt < ArtiodactylMLIn.txt

Continuous-time Markov models of trait evolution for discrete traits

Multistate and Discrete fit continuous-time Markov models to discrete character data. This model allows the trait to change from the state it is in at any given moment to any other state over infinitesimally small intervals of time. The rate parameters of the model estimate these transition rates (see Pagel (1994) for further discussion). The model traverses the tree estimating transition rates and the likelihood associated with different states at each node.

The table below shows an example of the model of evolution for a trait that can adopt three states, 0, 1, and 2. The q_{ij} are the transition rates among the three states, and these are what the method estimates on the tree, based on the distribution of states among the species. If these rates differ statistically from zero, this indicates that they are a significant component of the model. The main diagonal elements are constrained to be equal to minus the sum of the other elements in the row.

State	0	1	2
0		q 01	q 02
1	q 01		q 12
2	q 20	q 21	

Example of the model of evolution for a trait that adopts three states

For a trait that adopts four states, the matrix would have twelve entries, for a binary trait the matrix would have just two entries.

<u>Discrete</u> tests for correlated evolution in two binary traits by comparing the fit (loglikelihood) of two of these continuous-time Markov models. One of these is a model in which the two traits evolve independently on the tree. Each trait is described by a 2×2 matrix in the same format as the one above, but in which the trait adopts just two states, "0" and "1". This creates two rate coefficients per trait.

The other model, allows the traits to evolve in a correlated fashion such that the rate of change in one trait depends upon the background state of the other. The dependent model can adopt four states, one for each combination of the two binary traits (0,0; 0,1; 1,0; 1,1). It is represented in the program as shown below and the transition rates describe all possible changes in one state holding the other constant. The main diagonal elements are estimated from the other values in their row as before. The other diagonal elements are set to zero as the model does not allow 'dual' transitions to occur, the logic being that these are instantaneous transition rates and the probability of two traits changing at exactly the same instant of time is negligible. Dual transitions are allowed over longer periods of time, however. See Pagel, 1994 for further discussion of this model.

	-			
State	0,0	0,1	1,0	1,1
0,0		q 12	q 13	
0,1	q 21			q 24
1,0	q ₃₁			q ₃₄
1,1		q 42	q 43	

The values of the transition rate parameters will depend upon the units of measurement used to estimate the branch lengths in the phylogeny. If the branch lengths are increased by a factor 'c' the transition rates will be decreased by the same factor 'c'. This has implications for modelling the rate parameters in Markov chains as discussed below.

<u>Covarion model</u> BayesTraits implements the covarion model for trait evolution (Tuffley and Steel 1998). This is a variant of the continuous-time Markov model that allows for traits to vary their rate of evolution within and between branches. It is an elegant model that deserves more attention, although users may find it of limited value with comparative data.

The Generalised Least Squares model for continuously varying traits

<u>Continuous</u> analyses phylogenetically structured continuously varying data using a generalised least squares (GLS) approach that assumes a Brownian motion model of evolution (see (Pagel 1997, Pagel 1999))In the GLS model, non-independence among the species is accounted for

by reference to a matrix of the expected covariances among species. This matrix is derived from the phylogenetic tree. The model estimates the variance of evolutionary change (the Brownian motion parameter), sometimes called the 'rate' of change, and the ancestral state of traits at the root of the tree (alpha). It can also estimate the covariance of changes between pairs of traits, and this is how it tests for correlation.

The GLS approach as implemented in <u>Continuous</u> makes it possible to transform and scale the phylogeny to test the adequacy of the underlying model of evolution, to assess whether phylogenetic correction of the data is required, and to test hypotheses about trait evolution itself – for example, is trait evolution punctuational or gradual, is there evidence for adaptive radiation, is the rate of evolution constant.

Generalised Least Squares (GLS) and independent contrasts

The generalised least squares (GLS) method requires a number of computationally intensive calculations, including matrix inversions, Kronecker products and matrix multiplications. The time it takes to calculate a solution for GLS methods increases rapidly with the size of the tree making analysis on large trees hard. Independent contrast (Felsenstein 1973, Freckleton 2012) uses a restricted likelihood method, these methods are computationally efficient at the expense of not estimating some parameters, especially when using MCMC, see individual model description for information about which parameters are estimated. If speed is an issue, for data sets with hundreds or thousands of taxa, independent contrast should be favoured.

Hypothesis Testing: Likelihood ratios and Bayes Factors

BayesTraits does not test hypotheses for you but prints out the information needed to make hypothesis tests. These will be discussed in more detail in conjunction with the examples below, but here we outline the two kinds of tests most often used.

The likelihood ratio (LR) test is often used to compare two likelihoods derived from nested model (models that can be expressed such that one is a special or general case of the other). The likelihood ratio statistic is calculated as

LR= 2[log-likelihood(better fitting model) – log-likelihood(worse fitting model)]

The likelihood ratio statistic is asymptotically distributed as a χ^2 with degrees of freedom equal to the difference in the number of parameters between the two models. However, in some circumstances (Pagel 1994, Pagel 1997) the test may follow a χ^2 with fewer degrees of freedom.

Variants of the LR test include the Akaike Information Criterion and the Bayesian Information Criterion. We do not describe these tests here. They are discussed in many works on phylogenetic inference (see for example, (Felsenstein 2004)).

The LR, Akaike and Bayesian Information Criterion tests presume that the likelihood is at or near its maximum likelihood value. In a MCMC framework tests of likelihood often rely on Bayes Factors (BF). The logic is similar to that for the likelihood ratio test, except here we compare the marginal likelihoods of two models rather than their maximum likelihoods. The marginal likelihood of a model is the integral of the model likelihoods over all values of the models parameters and over possible trees, weighted by their priors. In practice this marginal likelihood is difficult to calculate and must be estimated. BayesTraits provides two methods to estimate the marginal likelihood, a harmonic mean estimate (Newton and Raftery 1994, Kass and Raftery 1995) and a stepping stone sampler (Xie, Lewis et al. 2011).

The harmonic mean estimate is calculated as the chain runs, after burn-in and is recorded in the "Harmonic Mean" column of the output, the harmonic mean is a running estimate so only the last value is used, the estimate of the harmonic mean will improve as the chain runs. Please note that because the harmonic mean is a running estimate, each sample of the chain does not produce an independent value and they cannot be analysed like other parameters. The harmonic mean can be very hard to estimate and may require multiple long runs for parameter rich analysis.

The stepping stone sampler estimates the marginal likelihood by placing a number of 'stones' which link the posterior with the prior, the stones are successively heated, forcing the chain from the posterior towards the prior, this provides an effective estimate of the marginal likelihood. The "stones" command, shortcut "st", is used to set the sampler, the command takes the number of stones and the length to run the chain on each stone. An example of setting the stones sampler is below, the command sets the sampler to use 100 stones and run each stone for 10,000 iterations.

stones 100 10000

The sampler runs after the chain has finished and produces a file with the extension "Stones.txt", the log marginal likelihood is recorded at the end of the file. Other information such as temperature, stones likelihood and marginal likelihood of each stone is also included but this is mainly for diagnostic purposes.

The marginal likelihood from the harmonic mean and stepping stone sampler are expressed on a natural log scale, these values can be converted into Log BF using the formula below. Raffety in (Gilks, Richardson et al. 1996) Pages 163–188, provides an interpretation of these values.

Log BF	Interpretation		
<2	Weak evidence		
>2	Positive evidence		
5-10	Strong evidence		
>10	Very strong evidence		

Log BF = 2(log marginal likelihood complex model – log marginal likelihood simple model)

Both the harmonic mean and stepping stone sampler estimate the marginal likelihood, these estimates are sensitive to a number of factors, including, priors, length of the chain and number of estimated paramiters. Care should be taken to ensure estimates are accurate and stable, multiple independent run should be used and the effect of using longer chains, more stones or sampling each stone for longer should be investigated. While two methods to calculate the marginal likelihood are provided the stepping stone sampler seems to produce more robust results especially when using models with higher dimensions. Model testing is a controversial topic with Bayesian analysis, and other options such as BIC, AIK, DIC may be considered.

Priors

When using the MCMC analysis method, the prior distributions of the parameters of the model of evolution must be chosen. Uniform or uninformative priors should be used if possible as these assume all values of the parameters are equally likely a priori and are therefore easily justified. Uniform priors can be used when the signal in the data is strong. But in a comparative study there will typically only be one or a few data points (unlike the many hundreds or thousands in a typical gene-sequence alignment) and so a stronger prior than a uniform may be required.

Priors are the soft underbelly of Bayesian analyses. The guiding principle is that if the choice of prior is critical for a result, you must have a good reason for choosing that prior. It is often useful to run maximum likelihood analyses on your trees to get a sense of the average values of the parameters. One option if a uniform with a wide interval does not constrain the parameters is to use a uniform prior with a narrower range of values, and this might be justified either on biological grounds or perhaps on the ML results. The ML results will not define the range of the prior but can give an indication of its midpoint.

NOTE: A rule of thumb when choosing a constrained or informed (non-uniform) prior is that if the posterior distribution of parameter values seems truncated at either the upper or lower end of the constrained range, then the limits on the prior must be changed.

The program allows uniform, exponential, gamma and beta distributed priors, specified as "exp", "gamma", "uniform", "beta". The uniform prior requires the user to specify a range, the exponential distribution always has its mode at zero and then slopes down, whereas the gamma can take a variety of uni-modal shapes or even mimic the exponential. The exponential prior is useful when the general feeling is that smaller values of parameters are more likely than larger ones. If the parameters are thought to take an intermediate value, a gamma prior with an intermediate mean can be used.

Priors are set using the prior command, the **Prior** command takes a parameter to set the prior for a distribution (uniform, exp, gamma or beta) and the parameters of the distribution. For example,

prior q01 exp 10

is used to set an exponential prior with a mean of 10 for the rate parameter q01

prior q10 uniform 0 100

sets the prior on q10 to a uniform 0 - 100

In many cases you will want to use the same prior on all parameters, the **PriorAll** command can be used to do this. It is identical to the prior command but does not take a parameter. For example,

priorAll exp 10

sets all prior to an exponential with a mean of 10

Because it can be difficult to arrive at suitable values for the parameters of the prior distributions, BayesTraits allows the use of a hyper-prior. A hyper-prior is simply a distribution – usually a uniform -- from which are drawn values to seed the values of the exponential or gamma priors. We recommend using hyperpriors as they provide an elegant way to reduce some of the uncertainty and arbitrariness of choosing priors in MCMC studies. For an example of selecting priors and using a hyper-prior see (Pagel, Meade et al. 2004)

When using the hyper-prior approach you specify the range of values for the uniform distribution that is used to seed the prior distribution. Thus, for example "HyperPriorAll exponential 0 10" seeds the mean of

the exponential prior from a uniform on the interval 0 to 10. "**HyperPriorAll** gamma 0 10 0 10" seeds the mean and variance of the gamma prior from uniform hyper priors both on the interval 0 to 10. For a full list of commands see the command list.

Burn-in and sampling in MCMC analysis

The burn-in period of a MCMC run is the early part of the run while the chain is reaching convergence. It is impossible to give hard and fast rules for how many iterations to give to burn-in. We often find that a minimum of 10,000 and seldom more than 1,00,000 is sufficient. The length of burn-in is set with the **burnin** command. During burn-in nothing is printed. With more complex models or larger trees often requiring longer burn-in periods.

Because successive iterations of most Markov chains are autocorrelated, there is frequently nothing to be gained from printing out each line of output. Instead the chain is sampled or thinned to ensure that successive output values are roughly independent. This is the job of the **sample** command. It instructs the program only to print out every nth sample of the chain. Choose this value such that the autocorrelation among successive points is low (this can be checked in most statistics programs or even Excel). For many comparative datasets, choosing every 1000th or so iteration is more than adequate to achieve a low autocorrelation.

The chain is run for 1010000 iterations by default, this can be changed with the **iterations** command, which takes the number of iterations to run for or -1 for an infinite chain, which can be stopped by holding Ctrl and pressing C.

The parameter proposal mechanism and mixing in MCMC analysis

Mixing

Mixing, the proportion a proposed change to a chain is accepted, is key to a successful MCMC analysis, MCMC proceeds by proposing changes to parameters. If proposed changes to a parameter are too large the likelihood will change dramatically, and at convergence many of the proposed changes will have a poor likelihood. This will cause the chain to have a low acceptance rate and the chain will mix poorly or even becoming stuck. The other side of the coin is, if small changes are proposed the likelihood does not change much, leading to a high acceptance rate, but the chain typically does not explore the parameter space effectively. An ideal acceptance rate is often between 20-40% when the chain is at convergence.

Parameter values can vary widely between data sets and trees, as the units data and branch lengths are in can vary orders of magnitude. This makes it very hard to find a universal proposal mechanism. An automatic tuning methods is used in BayesTraits to adapt the proposal mechanism to achieve an acceptance rate of 35%.

Monitoring Acceptance Rate

BayesTraits can produce a schedule file, using the "schedule" command, which is used to monitor how the chain is mixing, the file contains the schedule, the percentage of operators tried, followed by a header. The header shows the number of times an operator was tried and the percentage of time it was accepted, if auto tune is used the rate deviation values, acceptance rate for that parameter, the average acceptance for that iteration and the running mean acceptance rate is recorded. The schedule file should be reviewed to make sure the chain is mixing correctly.

Over parameterisation

Due to the statically nature of the methods it is possible to create over parameterised models, were too many parameters are estimated from not enough data. Indications of over parameterisation include, poorly estimated parameters, parameters trading off against each other, suboptimal likelihoods, and poor convergence / parameter optimisation. Model complexity can be reduced by combining parameters with the restrict command, or by using reverse jump MCMC and ensuring the ratio of parameters to data is not high.

Parameter restrictions

For multistate and discrete models the number of parameters increases roughly as square of the number of states, it is important to have sufficient data to estimate them. Multistate and discrete models allow parameters to be combined, reducing the number of free parameter. The restrict command (res) is used to restrict parameter, the command takes two or more parameter names, restricting all supplied parameters to the first

```
To restrict alpha2 to alpha1 use the following command Res alpha1 alpha2
```

```
To restrict all parameters, in an independent model, to alpha 1 use
Res alpha1 alpha2 beta1 beta2
Or
ResAll alpha1
```

Parameters can also be restricted to constants, including zero, in the same way Res alpha1 1.5 Or Res alpha1 alpha2 1.5

The unrestricted (UnRes) command can be used remove restrictions

Model testing (see above) can be used to test if a parameter is statistically justified, when rates are restricted the number of free parameters is reduced.

Reverse Jump MCMC

For a complex model the number of possible restrictions is large, and may be impossible to test. A reverse jump MCMC method (Green 1995) was developed to integrate results over model parameter and model restrictions, for a detailed description see (Pagel and Meade 2006).

The RevJump (RJ) command is used to select reverse jump MCMC, the command takes a prior and prior parameters. For example, the command below uses reverse jump with an exponential prior with a mean of 10. The second command uses reverse jump with a hyper exponential prior where mean of the exponential is drawn from a uniform 0 - 100

```
RevJump exp 10
Or
RJHP exp 0 100
```

For the general case

RevJump Prior Name Prior parameters

Or

RJHP Prior Name Prior parameters range

Where the prior name is "exp", "gamma", "uniform", "beta"

Parameter restrictions can be used in combinations with Reverse Jump, if you would like to set parameters to specific values or use a specific set of restrictions.

Multistate ML example

Start the program using the "Artiodactyl.trees" tree file and the "Artiodactyl.data" file. The following screen should be presented to you

Please select the model of evolution to use.
1) MultiState

Select 1 for the MultiState model

```
Please select the analysis method to use.
1) Maximum Likelihood.
2) MCMC
```

Select 1 for maximum likelihood analysis.

The default options will be printed, displaying basic information. This should always be checked to ensure it is what you expect.

Type run

The analysis will start. The options for the run will be printed followed by a header row. The header row is the

Header	Output
Tree No	The tree number, 1-500 for this data
Lh	Maximum likelihood value for the tree
qDG	The transition rate from D to G
qGD	The transition rate from G to D
Root P(D)	The probability the root is in state D
Root P(G)	The probability the root is in state G

For each tree in the sample a line of output will be printed. Once all trees have been analysed the program will terminate.

Multistate MCMC example

Start the program using the "Artiodactyl.trees" tree and "Artiodactyl.data" data file, select multistate (1) and MCMC (2). The default options will be printed.

Set all priors to an exponential with a mean of 10, and start the chain, using the commands

below.

PriorAll exp 10

Run

A header will be printed

Header	Output
Iteration	Current iteration of the chain
Lh	Current likelihood of the chain
Harmonic Mean	Running harmonic mean
Tree No	Current tree number
qDG	Transition rate from D to G
qGD	Transition rate from G to D
Root P(D)	Probability the root is in state D
Root P(G)	Probability the root is in state G

Followed by some output.

Iteration	Lh	Harmonic Mean	Tree No	qDG	qGD	Root P(D)	Root P(G)
11000	-7.93307	-7.93307	187	3.702561	2.5446	0.351023	0.648977
12000	-8.98846	-8.59398	495	3.120973	4.914959	0.475628	0.524372
13000	-8.37416	-8.52594	99	3.799383	4.489798	0.417393	0.582607
14000	-10.2806	-9.31231	95	17.07613	27.54498	0.499972	0.500028
15000	-10.7122	-9.78912	95	6.945588	5.865219	0.48436	0.51564
1006000	-8.94481	-9.90388	400	8.97661	8.407563	0.473517	0.526483
1007000	-8.53244	-9.90313	147	0.33644	1.477702	0.012116	0.987884
1008000	-8.03562	-9.90228	338	2.454093	3.334973	0.270869	0.729131
1009000	-8.41139	-9.90151	107	2.217407	4.183507	0.365974	0.634026
1010000	-9.72812	-9.90135	61	7.50541	9.538785	0.484114	0.515886

Output from the chain is tab separated and is designed to be using in program such as excel and JMP. Run to run output will vary and is depend on the random seed used.

Parameter restriction example

The previous example assumed that the transition rates from state D to G (qDG) and from state G to D (qGD) were different and both were estimated. To test if qDG and qGD are significantly different from each other, re-run the analysis restricting qGD to take the same value qDG. The same restrict command can be used in ML analysis.

```
PriorAll exp 10
Restrict qDG qGD
Run
```

The output should be very similar but the rate parameters (qDG and qGD) should take the same value each iteration. The significance of the test can be found by calculating a Bayes Factor using the either the harmonic means or the steppingstone sampler (see page 11).

If the marginal likelihood from the analysis where $qDG \neq qGD$ is -9.417807 and the marginal Lh is -8.261050 where qDG = qGD. The complex model is $qDG \neq qGD$, as it has one more parameter than the simple model, qDG = qGD. The Bayes Factor (BF) is given as

```
Log BF = 2(log marginal likelihood complex model – log marginal likelihood simple model)

Log BF = 2(-9.417807 - -8.261050)

Log BF = -2.313514

The BF is less than two so the simpler model should be favoured (see Gilks, Richardson et al.
```

(1996) or table on page 12)

Note: Values of the marginal likelihood calculated from the harmonic means and steppingstone sampler will vary between runs, depending on the random seed and how long the chain is run for, values are only for illustrative purposes. Harmonic mean estimate can vary from run to run. For reliability use multiple independent runs and allow them to run hundreds or even billions of iterations. Some authors are highly critical of harmonic means. The stepping stone (Xie, Lewis et al. 2011) estimation seem more stable. These values are only used to demonstrate basic model testing.

Ancestral state reconstruction Multistate / discrete

The AddMRCA and AddNode commands are used to reconstruct ancestral states in multistate and discrete models. The syntax for the two commands is similar. The commands take a tag which is used to identify the reconstructed node in the output, and a list of taxa names or taxa number which define the node. BayesTrees (http://www.evolution.reading.ac.uk/BayesTrees.html) is a graphics tree viewer which can be used to generate the command by clicking on the appropriate node. Using the "Artiodactyl.trees" tree and "Artiodactyl.data", select multistate and MCMC

The two commands below add a node to reconstruct, defined as the common ancestor of Porpoise Dolphin FKWhale and Whale. The node is called "Node1". The second command is identical but uses the taxa number instead of taxa names to define the node.

```
AddNode Nodel Porpoise Dolphin FKWhale Whale AddNode Nodel 5 6 7 8
```

run the program with the following commands

```
AddNode Nodel Porpoise Dolphin FKWhale Whale Run
```

Two new columns should be added to the output "Node1 P(D)" and "Node1 P(G)", these represent the probability of reconstructing a D or a G at Node1.

BayesTraits uses a sample of trees and some nodes may not be present in all trees, the node defined by Sheep, Goat, Cow, Buffalo and Pronghorn is only present in 58% of the trees.

```
PriorAll exp 10
Res qDG qGD
AddNode VarNode Sheep Goat Cow Buffalo Pronghorn
Run
```

The posterior probability of node reconstruction will not be present in some trees, some samples of the chain will recorded the ancestral sate as "--" because the node is not present in those trees.

The MRCA command reconstructs the Most Recent Common Ancestor, while a MRCA will be present in every tree it may not be the same node (see (Pagel, Meade et al. 2004) for more details). Rerun the analysis using MRCA.

```
AddMRCA VarNode Sheep Goat Cow Buffalo Pronghorn Run
```

Any number of nodes can be reconstructed in a single analysis without any effect on each other.

Fixing node values / fossilising

Internal nodes can be set to take a fixed value, if external information is available or to test if the value of one state is significant. The fossil command takes a node name, a state to fossilise it in and a list of taxa which define the node, nodes are found using the most recent common ancestor method. The command below fossilises a node defined by sheep, goats, cows, buffalo and pronghorn to state D.

Fossil Node01 D Sheep Goat Cow Buffalo Pronghorn

Be aware that fossilising nodes will have an effect on the models, by forcing a node to take a specific value the models parameters will be affected.

Fossilising states for discrete models requires a number instead of a state, as there are more combinations of fossil sates. The table below show the numbers and their corresponding states. X denotes the likelihood is left unchanged, - sets the likelihood to zero.

Number	0,0	0,1	1,0	1,1
0	Х	-	-	-
1	-	Х	-	-
2	-	-	Х	-
3	-	-	-	Х
10	Х	Х	-	-
11	Х	-	Х	-
12	Х	-	-	Х
13	-	Х	Х	-
14	-	Х	-	Х
15	-	-	Х	Х
20	Х	Х	Х	-
21	Х	Х	-	Х
22	Х	-	Х	Х
23	-	Х	Х	Х

Discrete examples

Discrete is used to test if two binary traits are correlated, significance is established by comparing the likelihoods of two models, one which assumes the traits evolve independently, with one which assumes the traits evolution is correlated. The examples focus on MCMC but ML can also be used. The examples use a sample of 500 primate trees ("Primates.trees") and a data set of two

binary traits, estrus advertisement and multi-male mating ("Primates.txt"). Two binary traits have 4 possible states, written as "0,0", "0,1", "1,0" and "1,1".

Discrete independent

The independent model assumes the two traits evolve independently, e.g. the transition from $0 \rightarrow 1$ in the first trait is independent of the state of the second trait. The independent model has 4 rate parameters, alpha1, beta1, alpah2 and beta2.

Ρ	arameter	Symbol	Tr	ait 1	Transitions	
	alpha1	α1		1	$0 \rightarrow 1$	
	beta1	β1		1	$1 \rightarrow 0$	
	alpha2	α2		2	$0 \rightarrow 1$	
	beta2	β 2		2	$1 \rightarrow 0$	
		0,0	0,1	1,0	1,1	_
-	0,0	-	α_2	α_1	0	-
	0,1	βz	-	0	α_1	
	1,0	β_1	0	-	α_2	
	1,1	0	β_1	β₂	-	

Start BayesTraits with the tree file "Primates.trees" and data file "Primates.txt", select the independent model (2) and MCMC analysis (2). Set all the priors to an exponential with a mean of 10 and run the analysis, this mean was found by analysing the tree using ML and studding the resulting parameter rate estimates, using the following commands.

```
PriorAll exp 10
Run
```

The output will be similar to the multistate analysis, the header will contain.

Header	Output
Iteration	Current iteration of the chain
Lh	Current likelihood of the chain
Harmonic Mean	Running harmonic mean
Tree No	Current tree number
alpha1	The alpha1 transition rate
beta1	The beta1 transition rate
alpha2	The alpha2 transition rate
beta2	The beta2 transition rate
Root – P(0,0)	Probability the root is in state 0,0
Root – P(0,1)	Probability the root is in state 0,1
Root – P(1,0)	Probability the root is in state 1,0
Root – P(1,1)	Probability the root is in state 1,1

Discrete dependent

The dependent model assumes that the traits are correlated and the rate of change in one trait is dependent on the state of the other. The dependent model has 8 parameters, q12, q13, q21, q24, q31, q34, q42 and q43. Double transitions from state 0,0 to 1,1 or from 0,1 to 1,0 are set to zero,

Parameter	Dependent on	Trait	Transitions
q12	Trait 1 = 0	2	$0 \rightarrow 1$
q13	Trait 2 = 0	1	$0 \rightarrow 1$
q21	Trait 1 = 0	2	$1 \rightarrow 0$
q24	Trait 2 = 1	1	$0 \rightarrow 1$
q31	Trait 2 = 0	1	$1 \rightarrow 0$
q34	Trait 1 = 1	2	$0 \rightarrow 1$
q42	Trait 2 = 1	1	$1 \rightarrow 0$
q43	Trait 1 = 1	2	$1 \rightarrow 0$

	0,0	0,1	1,0	1,1
0,0	-	q _{1,2}	q _{1,3}	0
0,1	q _{2,1}	-	0	q _{2,4}
1,0	q _{3,1}	0	-	q _{3,4}
1,1	0	q _{4,2}	q _{4,3}	-

Start BayesTraits with the tree file "Primates.trees" and data file "Primates.txt", select the dependent model (3) and MCMC analysis (2). Set all the priors to an exponential with a mean of 10 and run the analysis, using the following commands.

PriorAll exp 10

Run

The output will be very similar to the independent model except that the dependent parameters are estimated instead of the independent.

To test if the traits are correlated calculate a Bayes Factor (see page 11) between the independent and dependent models. If the independents models marginal likelihood is -45.51 and the dependent harmonic mean is -42.47

Log BF = 2(log marginal likelihood complex model – log marginal likelihood simple model) Log BF = 2(-42.47 - -45.51) Log BF = 6.08

The Log BF of 6 suggests there is strong evidence for correlated evolution. Marginal likelihoods vary between runs and it is important to get a stable estimate by using multiple independent runs and long chains.

Reverse Jump MCMC and model reduction

Given the size of the data and complexity of the model not all parameters may be statistically distinguishable. The previous parameter restriction example demonstrated how a model could be simplified by setting parameters equal to each other and how to test if restrictions were significant. There are 51 possible restrictions for the independent model and over 21,000 for the dependent model, which would take a long time to test. Reverse jump MCMC (RJ-MCMC) offers an alternative by integrating results over the model space, weighting naturally by their probabilities, allowing the user to select viable models and parameters, see Pagel, Meade et al. (2004) for more information.

The reverse jump command takes a prior as a parameter, one prior must be applied to all parameters, the command below uses an RJ MCMC model with an exponential prior with a mean of 10

RJ exp 10

RJ MCMC can also be used with a hyper-prior.

RJHP exp 0 100

Run the primates data and tree, with the dependent model and MCMC analysis, using the commands below

```
RJ exp 10
Run
```

The output will contain 4 new columns.

Header	Output	
No Of Parameters	Number of parameters	
No Of Zero	Number of parameters set to zero	
Model string	A model string showing parameter restrictions	
Dep / InDep	A flag showing if the model is dependent (D)	
	or independent (I)	

Model strings are used to characterise the models restrictions, the string start with ' and is followed by numbers indicating which parameters are in which groups or a Z if the parameters have been restricted to zero. For example the modelling string for a dependent model will have 8 components one for each parameter, the model string "'1 Z 0 0 0 1 1 Z", has two parameters and two rates set to zero. The first group consists of the 1st, 6th and 7th parameters (q12, q34 and q42), the second group is formed of the 3rd, 4th and 5th parameters (q21, q24 and q31), and the 2nd and 8th parameter is set to zero. This can be checked against the parameter estimates.

To test if a data set is correlated running an independent model using RJ MCMC and a dependent model using RJ MCMC.

Covarion model

BayesTraits implements a basic on / off covarion model as described by (Tuffley and Steel 1998), the model requires one additional parameter the switching rates between the on / off states. The model allows the rate of evolution to vary through the tree. The "CV" command is used to activate the covarion model, two additional columns will be included in the output, "Covar On to Off" and "Covar Off to On". The switching rate between the on and off states will be the same.

Continuous: Random Walk (Model A) ML

Start BayesTraits with the tree file "Mammal.trees" and data file "MammalBody.txt", the tree file is a sample of 50 mammal trees and the data is there corresponding body size, the trees and data are for illustrative purposes and are not accurate or a good sample. Select model A (4) and maximum likelihood analysis (1), start the analysis with

Run

HeaderOutputTree NoThe tree number, 1-50 for this dataLhMaximum likelihood value for the treeAlpha 1The phylogenetically correct mean of the data, also the estimated root valueSigma^2 1The phylogenetically corrected variance of the data

Basic information will be printed followed by a header

Continuous: Random Walk (Model A) MCMC

Start BayesTraits with the tree file "Mammal.trees" and data file "MammalBody.txt", select Model A (4) and MCMC (2), stat the analysis with

Run

Header	Output
Iteration	Current iteration of the chain
Lh	Current likelihood of the chain
Harmonic Mean	Running harmonic mean
Tree No	Current tree number
Alpha Trait 1	The phylogenetically correct mean of the data, also the estimated root value
Sigma^2 1	The phylogenetically corrected variance of the data

Basic information will be printed followed by a header

Testing trait correlations: continuous

To test if two traits are correlated, the results from two analysis are compared, one in which a correlation is assumed (the default) and one where the correlation is set to zero. Run an analysis using the tree file "Mammal.trees" and a data file "MammalBrainBody.txt", containing brain and body size data. Select model A (4) and MCMC analysis (2), stat the analysis with

Run

Basic information will be printed followed by a header

Header	Output	
Iteration	Current iteration of the chain	
Lh	Current likelihood of the chain	
Harmonic Mean	Running harmonic mean	
Tree No	Current tree number	
Alpha 1	The phylogenetically correct mean of the first trait	
Alpha 2	The phylogenetically correct mean of the second trait	
Sigma^2 1	The phylogenetically corrected variance of the first trait	
Sigma^2 2	The phylogenetically corrected variance of the second trait	
R Trait 1 2	R correlation between trait 1 and trait 2	

Rerun the analysis but force the correlation to be zero using the TestCorrel (TC) command.

TestCorrel Run

The output should be similar except the "R Trait 1 2" value will be 0. The significance of the correlation can be tested by computing a Bayes Factor between the two runs. If the analysis allowing a correlation produced a marginal likelihood of -80.845 and the analysis with the correlation fixed to zero gave a marginal likelihood of -140.58, this would lead to a log Bayes Factor of 119.47, suggesting they are highly correlated.

Continuous: Directional (Model B) MCMC

The directional model can be used to test if there is a directional change in a traits evolution, by testing if a trait is correlated with the root to tip distance of the taxa. Model B cannot be used with ultrametric trees as there is no root to tip variation between taxa. A fictional data set "MammalModelB.txt" can be used to test if there is a significant directional trend by preforming a

model test between Model A and Model B. If Model B is significant these is signal for a directional trend in the data.

Continuous: Regression

The continuous regression model is used to perform regression analysis, test trait significance and predict unknown values. The regression model takes two or more traits, the first trait is assumed to be the dependent variable. MammalBrainBodyGt.txt is a dataset of mammal brain, body and gestation time. Run BayesTraits with the "Mammal.trees" tree file and "MammalBrainBodyGt.txt" data. Select the regression model (6) and MCMC (2) and run the analysis. The header will contain.

Header	Output	
Iteration	Current iteration of the chain	
Lh	Current likelihood of the chain	
Harmonic Mean	Running harmonic mean	
Tree No	Current tree number	
Alpha	Intercept	
Beta Trait 2	Regression coefficient for trait 2	
Beta Trait 3	Regression coefficient for trait 3	
Var	Brownian motion variance	
R^2	R^2	
SSE	Sum of squared error	
SST	Total sum of squared	
s.e. Alpha	Standard error Alpha	
s.e. Beta-2	Standard error Beta-2	
s.e. Beta-3	Standard error Beta-3	

Testing trait significance

There are a number of ways to test if a trait is significant in the regression model, the first is to compare marginal likelihood (MCMC) or likelihood ratios (ML) from runs with and without the trait. The second is the ratio of the time the regression coefficient is crosses the zero point, if a regression coefficient is well supported it will not switch from positive to negative, or vice versa.

Continuous: Estimating unknown values

Continues models can be used to estimate unknown values on the tree, internal nodes or tip. Estimating unknown values is a two-step process, first a distribution of models is estimated from available data, secondly the models are used to estimate unknown values. The two step process prevent estimated data from affecting the model parameters. Estimating unknown values can be used with model A, model B and the regression model but only using MCMC.

The "SaveModels" command is used to save models to a specified file, the "LoadModels" command is used to load the models into BayesTraits. The same model parameters, including tree transformations, has to be specified when creating a model file and when estimating unknown values, only very basic error checking is implemented.

Example: Estimating unknown values internal nodes

Start BayesTraits with the "Mammal.trees" file and "MammalBody.txt" data. Select model A (4) and MCMC analysis (2). Save the models and run the analysis with the commands below, the models will be saved into a file called "MamBodyModels.bin"

```
SaveModels MamBodyModels.bin Run
```

Once the program has finished a file called "MamBodyModels.bin" will be created.

To estimate data, start BayesTraits with the same tree and data files, select model A (4) and MCMC analysis (2). The AddMRCA command is used to estimate internal node values, it takes a node label used to identify the node in the output and a list of taxa which define the node. The commands below load the model file, reconstruct a node called "Node-01" defined by five taxa, reconstruct a node called "Node-02" defined by four taxa, and run the analysis.

```
LoadModels MamBodyModels.bin
AddMRCA Node-01 Whale Hippo Llama Ruminant Pig
AddMRCA Node-02 Mouse Rat Hystricid Caviomorph
Run
```

The output header will contain

Header	Output	
Iteration	Current iteration of the chain	
Lh	Current likelihood of the chain	
Harmonic Mean	Running harmonic mean	
Tree No	Current tree number	
Model No	Current model number from the model file	
Alpha Trait 1	Phylogenetic mean	
Sigma^2 1	Brownian motion variance	
Est Node-02 - 1	Estimated values for Node-02 trait 1	
Est Node-01 - 1	Estimated values for Node-01 trait 1	

Example: Estimating unknown values for tips

Data for taxa, as well as internal nodes can be estimated. A data file "MammalBrainBodyNoTapir.txt" has been created with the data for tapir missing '-'. Run BayesTraits with the "Mammal.trees" tree file and "MammalBrainBodyNoTapir.txt" data file. Select the regression model (6) and MCMC analysis (2). Use the command below to save the models to a file and run the analysis

```
SaveModels MamRegModels.bin Run
```

A data file "MammalBrainBodyPredTapir.txt" which contains the tapir body size but with the brain size set to "?", a question mark in the data is used to indicate the value should be estimated. Run an analysis using the "Mammal.trees" tree file and "MammalBrainBodyPredTapir.txt" data file. Select the regression model (6) and MCMC analysis (2). Use the command below to load the models and run the analysis

```
LoadModels MamRegModels.bin Run
```

The output will contain a column labelled "Est Tapir – Dep", with the predicted tapir brain size, the predicted brain size should be roughly 2.1 ± 0.15 , the actual brain size is 2.2.

Independent contrast

BayesTraits implements a range of independent contrast models, independent contrast offers a significantly faster alterative to Generalised Least Squares (GLS) methods for large data sets. Independent contrast models can be run using MCMC or ML. Start BayesTraits with the tree file "Mammal.trees" and data file "MammalBrainBody.txt". Select Independent contrast (7) and MCMC (2) and run the analysis. The independent contrast model assumes sites are independent, e.g the covariance is set to zero. The output will contain

Header	Output	
Iteration	Current iteration of the chain	
Lh	Current likelihood of the chain	
Harmonic Mean	Running harmonic mean	
Tree No	Current tree number	
Alpha 1	Phylogenetic mean of the first trait	
Alpha 2	Phylogenetic mean of the second trait	
Sigma^2 1	Brownian motion variance for the first trait	
Sigma^2 2	Brownian motion variance for the second trait	

Variable rates model

The variable rates model allows the rate of change to vary threw time and identifies areas of the tree where the rate of evolution differed significantly, for an in-depth description see (Venditti, Meade et al. 2011). The variable rates model uses RJ MCMC to identify areas of the tree in which the rate of evolution varies significantly. The model only works with a single tree and requires MCMC analysis. A tree "Marsupials.trees" of roughly 250 marsupials and there body sizes "Marsupials.txt" is included. Start BayesTraits with the tree and data file, select Independent contrast (7) and MCMC (2), run the variable rates analysis with the commands below.

VarRates Run **The log file will contain**

Header	Output	
Iteration	Current iteration of the chain	
Lh	Current likelihood of the chain	
Harmonic Mean	Running harmonic mean	
Tree No	Current tree number	
Alpha 1	Phylogenetic mean of the first trait	
Sigma^2 1	Brownian motion variance for the first trait	
No VarRates	Number of areas of the tree with variables rates	

Other data files are created, "Marsupials.txt.PP.trees" contains the trees scaled by the rate of change, areas which are stretched have an increased rate of change, areas which are shrunken have a decreased rate. The file "Marsupials.txt.PP.txt" contrails a detailed description of the

changes, the format of the file is, line 1, number of taxa, followed by a unique taxa ID and taxa name. The second part is the number of internal nodes, followed by a list of internal nodes consisting of a unique node ID, branch length (-1 for root), number of taxa which define the node and the list of taxa ID. The third section details the results of the chain, the columns are

Header	Output
lt	Iteration of the chain
Lh	Likelihood of the chain
Lh + Prior	Likelihood + prior
No Pram	Number of change to rate of the tree
Alpha	Estimated phylogenetic mean
Sigma^2	Brownian motion
Alpha Scale	Scale of the prior (unchanging, for diagnostics only)

For each change of rate of the tree (No Param) there is

Header	Output
Node ID	The node if the change is on
Scale	The scale of the change
Crate It	The iteration the change was created on
Node / Branch	If the change is a node or branch scale

The .PP.txt file is designed to be computer read, post processing tools will shortly be made available to extract useful information from the file.

Independent contrast: Correlation

The independent contrasts correlation model allows correlations to be tested between two traits using the independent contrast framework.

Start BayesTraits with the tree file "Mammal.trees" and data file "MammalBrainBody.txt". Select Independent Contrast: Correlation (8) and MCMC (2) and run the analysis. The default analysis assume the trait are correlated, the output will contain

Header	Output
Iteration	Current iteration of the chain
Lh	Current likelihood of the chain
Harmonic Mean	Running harmonic mean
Tree No	Current tree number
Alpha 1	Phylogenetic mean of the first trait
Alpha 2	Phylogenetic mean of the second trait
Sigma^2 1	*Brownian motion variance for the first trait
Sigma^2 2	*Brownian motion variance for the second trait
CoVar 1-2	*The covariance between the first and second trait

To test a correlation between two traits rerun the analysis using the "TestCorrel" (TC) command to set the co-variance to zero and calculate a Bayes Factor.

* Due to the independent contrast framework the variance and covariance parameter are set to their maximum likelihood values even when using MCMC.

Independent contrast: regression

The independent contrast models supports simple and multiple regression, using ML and MCMC. The first trait is assumed to be the dependent variable, subsequent traits are assumed to be the independent variables. Run an analysis using the tree file "Mammal.trees" and the data file "MammalBrainBodyGt.txt". Select "Independent Contrast: Regression", 9 and MCMC 2, and run the analysis. The log file will contain

Header	Output
Iteration	Current iteration of the chain
Lh	Current likelihood of the chain
Harmonic Mean	Running harmonic mean
Tree No	Current tree number
Alpha	*Analytical value of Intercept
Beta 1	Regression coefficient for trait 2
Beta 2	Regression coefficient for trait 3

* Due to the independent contrast framework the maximum likelihood intercept value is used, calculated from the regression coefficient, even when using MCMC, it is not an estimated value.

Discrete: Covarion

The discrete covarion model removes the assumption that a trait is either correlated or not threw out the tree. It simultaneously fits both an independent and dependent model and uses a covarion method to switch between them.

Start BayesTraits with the primates tree and dataset, select the discrete covarion model (10), MCMC (2) and run the analysis.

Header	Output
Iteration	Current iteration of the chain
Lh	Current likelihood of the chain
Harmonic Mean	Running harmonic mean
Tree No	Current tree number
alpha1	The independent alpha1 transition rate
beta1	The independent beta1 transition rate
alpha2	The independent alpha2 transition rate
beta2	The independent beta2 transition rate
q12	The dependent q12 transition rate
q13	The dependent q13 transition rate
q21	The dependent q21 transition rate
q24	The dependent q24 transition rate
q31	The dependent q31 transition rate
q34	The dependent q34 transition rate

q42	The dependent q42 transition rate
q43	The dependent q43 transition rate
qDI	The switching rate between the dependent and independent models
qID	The switching rate between the independent and dependent models
Root - I P(0,0)	Probability the root is in independent state 0,0
Root - I P(0,1)	Probability the root is in independent state 0,1
Root - I P(1,0)	Probability the root is in independent state 1,0
Root - I P(1,1)	Probability the root is in independent state 1,1
Root - D P(0,0)	Probability the root is in dependent state 0,0
Root - D P(0,1)	Probability the root is in dependent state 0,1
Root - D P(1,0)	Probability the root is in dependent state 1,0
Root - D P(1,1)	Probability the root is in dependent state 1,1

The discrete covarion model supports RJ MCMC, to reduce the large number of parameters required.

Tree transformations, kappa, lambda, delta, OU

BayesTraits supports a number of tree transformations including, kappa (κ), lambda (λ), delta (δ) and Ornstein Uhlenbeck (OU) for both continuous and independent contrast models. These scaling parameters allow tests of the tempo, mode, and phylogenetic associations of trait evolution. Kappa, lambda, and delta take the value 1.0 by default, OU take the value 0. These values correspond to assuming that the phylogeny and its branch lengths accurately describe a constant-variance random walk model A or B. However, if trait evolution has not followed the topology or the branch lengths, these values will depart from 1.0. When they do, incorporating them into the analysis of the data (e.g., when estimating the correlation between two traits) significantly improves the fit of the data to the model.

The kappa parameter differentially stretches or compresses individual phylogenetic branch lengths and can be used to test for a punctuational versus gradual mode of trait evolution. Kappa > 1.0 stretches long branches more than shorter ones, indicating that longer branches contribute more to trait evolution (as if the rate of evolution accelerates within a long branch). Kappa < 1.0 compresses longer branches more than shorter ones. In the extreme of Kappa = 0.0, trait evolution is independent of the length of the branch. Kappa = 0.0 is consistent with a punctuational mode of evolution.

The parameter delta scales overall path lengths in the phylogeny - the distance from the root to the species, as well as the shared path lengths. It can detect whether the rate of trait evolution has accelerated or slowed over time as one moves from the root to the tips, and can find evidence for adaptive radiations. If the estimate of Delta < 1.0, this says that shorter paths (earlier evolution in the phylogeny) contribute disproportionately to trait evolution - this is the signature of an adaptive radiation: rapid early evolution followed by slower rates of change among closely related species. Delta > 1.0 indicates that longer paths contribute more to trait evolution. This is the signature of accelerating evolution as time progresses. Seen this way, delta is a parameter that detects differential rates of evolution over time and re-scales the phylogeny to a basis in which the rate of evolution is constant.

The parameter lambda reveals whether the phylogeny correctly predicts the patterns of covariance among species on a given trait. This important parameter in effect indicates whether one of the key assumptions underlying the use of comparative methods - that species are not independent - is true for a given phylogeny and trait. If a trait is in fact evolving among species as if they were independent, this parameter will take the value 0.0 and indicate that phylogenetic correction can be dispensed with. A lambda value of 0.0 corresponds to the tree being represented as a star or big-bang phylogeny. If traits are evolving as expected given the tree topology and the random walk model, lambda takes the value of 1.0. Values of lambda = 1.0 are consistent with the constant-variance model (sometimes called Brownian motion) being a correct representation of the data. Intermediate values of lambda arise when the tree topology over-estimates the covariance among species.

The value of lambda can differ for different traits on the same phylogeny. If the goal is to estimate the correlation between two traits then lambda should be estimated while simultaneously estimating the correlation. If, on the other hand, the goal is to characterise traits individually, a separate lambda can be estimated for each.

The Ornstein Uhlenbeck (OU) transform has traditionally been associated with stabilising selection, with the OU parameter measures the strength of a return to a theoretical optimum (Hansen 1997) this may be due to other factors and care should be taken when using and interpreting OU results. OU parameter values of zero correspond to the default values, parameter values >0 are evidence of an OU process. The OU module should only be used with ultrametric trees, the correction for non-ultrametric trees will be available soon.

Parameter	Action	0	<1	1	>1
lambda	Assess contribution of phylogeny	Star phylogeny (species independent)	phylogenetic history has minimal effect	default phylogeny	not defined
kappa	Scale branch lengths in tree	punctuational evolution	stasis in longer branches	default gradualism	longer branches more change
delta	Scale total path (root to tip) in tree	not defined	temporally early change important (adaptive radiation)	default gradualism	temporally later change (species specific adaptation)
OU		default	>0	evidence of an	OU process

Three scaling parameters and their interpretation when applied to trait evolution on a phylogeny

All three parameters can be estimated using ML or MCMC, the syntax for the three parameters is the same, either the scaling parameter on its own to toggle its estimation (they are not estimated by default, set to 1.0) or the scaling parameter followed by a number to fix the parameter to a given value.

The first command estimates lambda, the second fixes it to 0.5 Lambda 0.5

The first command estimates kappa, the second fixes it to 0.5 $_{\rm Kappa}$

Kappa 0.5

The first command estimates delta, the second fixes it to 0.5 Delta Delta 0.5

The first command estimates OU, the second fixes it to 0.5 $_{\rm OU}^{\rm OU}$ 0.5

Model testing can be used to determine if a transform is significant or if a value of transform is significant, e.g. is lambda significantly different from 0.

Alternative Builds

Precision Quad

Computers represent number using a specific number of bits, typically floating point numbers used in *BayesTraits* are double precision requiring 64 bits. As the amount of data available to store a number is limited, only number within a specified range can be stored, double precision supports numbers as small as 2.2×10^{-308} or rough -708 when on a log scale. For discrete and multistate models each site cannot have a likelihood less than -708 log units or the likelihood underflows and is rounded to zero, creating a likelihood of- ∞ when logged. This is partially a problem for large trees (many hundreds or thousands of taxa) or when analysis data with a large number of states.

To solve this problem *BayesTraits* supports quadruple precision number, allowing for smaller likelihood values to be calculated but requiring longer runtimes, if you are experiencing an underflow problem try using a quadruple precision version of *BayesTraits*, these are labelled quad. Quadruple precision supports likelihoods as low as roughly -11355 log units per site, this may not be enough for very large trees. A version of *BayesTraits* with high precision than quadruple is available if required.

Multi Core

The Multi Core version of *BayesTraits* allows the likelihood calculation, the computationally intensive part of the program, to be run over multiple cores for multistate and discrete models, this can speed up the runtime of the program. The speed increase is data set and model dependent, speed improvements are uncommon on tress of less than 1000 taxa, as the time needed for coordinating the cores out ways there speed improvement. For larger trees the speed improvement is significant, especially if quadruple precision is needed, see above. As the OpenMP version will not always give a speed increases, it is worth checking the performance of the serial versions of *BayesTraits* with the Multi Core version. The "cores" command can be used to specify the number of cores to use, see command list below, this can be used to tune performance.

OpenCL

OpenCL allows computationally intestine operations to be performed on graphics hardware, which can give large speed increases, especially for big trees, typically thousands of taxa. To use OpenCL you need three things, graphics hardware that supports OpenCL, an OpenCL driver installed and the OpenCL version of *BayesTraits*.

OpenCL graphics hardware

The Khronos group, who oversee the standard, keep a list of OpenCL compatible hardware (<u>http://www.khronos.org/conformance/adopters/conformant-products/</u>). Typically any AMD and NVIDA graphics cards purchased in the last couple of year should support OpenCL but please check. Each month hundreds of graphics cards are released with different processors, memory and interfaces. The web site CompuBench (<u>www.compubench.com</u>) provides a compressive comparison of OpenCL performance for different graphics cards.

It is possible to get over a 40 fold speed increases compared to Version 1, the performance increase is heavily depend on a number of factors, including tree and data size, model type and

complexly, the underlying hardware both graphics cards and system, and the drivers and operating system used. As with the OpenMP version of *BayesTraits* it is possible for the OpenCL version to run slower.

OpenCL driver

An OpenCL driver is software which provides a level of abstraction between the hardwere and software, allowing a program written using OpenCL to run on a larger number of different hardware platforms. The OpenCL driver is supplied by the hardware manufacture, normally AMD or NVIDIA, on some systems it comes with the graphics driver and does not need to be installed separately. If the OpenCL version of BayesTraits fails to start or reports and error on before the run starts, suitable graphics hardware or OpenCL driver may be missing.

Common problem / Frequently Asked Questions

1) Problems starting the program

- Q) Double clicking on the program does not work.
- A) BayesTraits is run from the command line and not by double clicking on it. See "Running BayesTraits" section.

2) Common tree and data errors

- A) Tree must be in nexus format with a valid translate block. Use the example files as a template.
- B) Tree descriptions must have number and not taxa name in them.
- C) Trees must be rooted.
- D) Trees and data must be encoded using ASCII format not Unicode
- E) The error "Could not load data for taxa X", this error is caused by a taxa being specified in the tree file but not in the data file. Check spelling and taxa numbers
- F) The error "Tree file does not have a valid nexus tag." Is because a nexus tag is not found in the tree file. Possible causes are specifying the data file before the tree file.

3) "Memory allocation error in file ..."

Error message "Memory allocation error in file ...", is a catch all memory allocation error, the two main causes of memory allocation errors are running out of memory, this can be due to too many trees in the tree file or a complex memory intensive models. Check the programs memory usage, if you have a 64 bit OS use the 64 bit version of the program. Try running the program with a smaller number of trees and simpler models. The second cause of the crash is due to programming errors, if you believe this is the case, please send along the tree file, data file and set of command used.

4) Chain is not mixing between trees.

This problem can be caused when one tree's likelihood is significantly better than other trees in the sample. Trees are sample in proportion to their likelihood, if one is much better than the rest it will be sample much more often. This can be a particular problem with a large number of trees when the topology is poorly supported, a chance combination create a much better likelihood preventing the chain from mixing. To test if this is the problem run the sample using ML, this will determine if the tree which the chain gets stuck on has the best likelihood. Two options are available, the first is to remove the tree from the sample if you believe it is anomalous for some reason. The second is to use the Equal Tree command to force the chain to spend an equal amount of time on each tree. The equal tree command will produce a separate posterior sample of rates, parameters and ancestral states per tree, instead of a single set integrated over the sample of trees.

5) Cannot find a valid set of starting parameters.

A valid set of parameters to start the chain cannot be found. Two main causes of this error are

i) The size of the tree and / or size of the model cause an underflow error and so a valid starting likelihood cannot be found. See precision section (under information

for a full description and solution), this is a particular problem with large trees and discrete / multistate models.

ii) The model may be invalid, the combination of restrictions, priors or data may produce an invalid model, for example setting all the transition rates to zero. Try using simpler models, restricting the number model to a single transition rate and slowly building to more complex models.

6) Too many free parameters to estimate

The number of free parameters is limited to 25 or fewer for maximum likelihood. Likelihood methods allow parameters to be estimated from data, comparative methods data can be limited, typically consisting of a one or more site for a number of taxa. This limits the number of parameters which can be accurately estimated form the data. Accurately estimating 25 or more parameters would require a vast amount of data. So the number of free parameters under ML is limited, if you believe your data can support more parameters please contact the authors for this limitation to be removed.

7) Run to run variations

It is important to run the same analysis a number of times, to check for convergence when using MCMC and to ensure that optimal parameter values have been found when using ML. Large differences between runs is often a warning sign, commonly this is because of to many parameters for the data to support or the parameters create a deceptive likelihood surface, for MCMC this can prevent the chain from converging and / or mixing, for ML different runs may produce different results. Multiple runs can help identify if this is a problem, for ML analysis increasing the number of maximum likelihood tries (MLT) per tree is important, the default of 10 gives a balance between speed and accuracy, increasing this number can help if there is large run to run variations but also increases the run time.

Command List

Command: Purpose: Shortcut: Parameters:	// Add a comment. # None
Example:	# This is a comment.
Command: Purpose: Shortcut: Parameters: Example:	AddMRCA To reconstruct an internal node using the most recent common ancestor approach MRCA A node name and a list of taxa names or number that define a node to reconstruct. AddMRCA Node1 Taxa1 Taxa2 Taxa3 MRCA Node1 1 2 3 4
Command: Purpose: Shortcut: Parameters: Example:	AddNode To reconstruct an internal node. AddN A node name and a list of taxa names or number that define a node to reconstruct. AddNode Node1 Taxa1 Taxa2 Taxa3 AddN Node1 1 2 3 4
Command: Purpose: Shortcut: Parameters: Example:	AlphaZero Sets the intercept to zero AZ None AlphaZero
Command: Purpose: Shortcut: Parameters: Example:	BurnIn To set the number of iterations to burn the MCMC chain in for, use -1 for an infinite chain. BI An integer BurnIn 50000 BI -1
Command: Purpose: Shortcut: Parameters: Example:	CapRJRates Cap the maximum number of reverse jump rates to use Cap An integer, >0 CapRJRates 2 Cap 1
Command: Purpose: Shortcut: Parameters: Example:	Cores Set the number of cores to use, only available with Multi Core builds and discrete / multistate models. cor An integer, >1 cores 2 cor 4

Command: Purpose: Shortcut: Parameters: Example:	CoVarion Turn on/off the convarion model CV None CoVarion CV
Command: Purpose: Shortcut: Parameters: Example:	DataDev Set the deviation parameter to used when perturbing estimated data. This value should be automatically set, modifying the value is not recommended. DD A number, >0 DataDev 2.0 DD 01
Command: Purpose: Shortcut: Parameters: Example:	Delta Estimate delta or set it to a fixed value DL None to estimate delta, or a number to fix it to a value Delta Delta 0.5
Command: Purpose: Shortcut: Parameters: Example:	EqualTrees Force the chain to spend an equal amount of time on each tree in the sample. This results in a separate posterior distribution pre tree. EQT Number of iterations to burn each tree in for. EqualTrees 20000
Command: Purpose: Shortcut: Parameters: Example:	EvenRoot Set midpoint rooting for the sample. ER None EvenRoot
Command: Purpose: Shortcut: Parameters: Example:	Exit Exit BayesTraits without running the analysis Quit None Exit
Command: Purpose: Shortcut: Parameters: Example:	Fossil Fix an internal node to a specific value FO A name, the value to fossilise the node in and a list of taxa which define the node. See the fixing node values section above for more information. Fossil AnsNode D 12 13 14 15 16 17 Fossil Base 2 Hylobates_gabriellae Hylobates_leucogenys Hylobates_concolor

Command: Purpose: Shortcut: Parameters: Example:	Gamma Estimate or fix gamma rate heterogeneity. GA The number of gamma categories and an option value to fix the parameter Gamma 4 Gamma 4 0.5
Command: Purpose: Shortcut: Parameters: Example:	Help Print a list of commands, not all are valid / working he none Help
Command: Purpose: Shortcut: Parameters: Example:	HyperPrior Set a hyper prior on a parameter HP A parameter name, a distribution name, and range to draw each parameter from HyperPrior q01 exp 0 100 HP q10 gamma 0 100 0 100
Command: Purpose: Shortcut: Parameters: Example:	HyperPriorAll Set all priors to a common hyper prior HPAll A distribution name and range to draw each parameter from HyperPriorAll Beta 0 100 0 50 HPAll Exp 0 200
Command: Purpose: Shortcut: Parameters: Example:	Info Print current options in None Info
Command: Purpose: Shortcut: Parameters: Example:	Iterations Set the number of iterations to run the chain for IT The number of iterations to run the chain for, or -1 for an infinite chain, use Ctrl+C for termination. Iterations 1000000 IT -1
Command: Purpose: Shortcut: Parameters: Example:	Kappa Set the kappa scaling parameter KA None to estimate kappa, or a number to fix it to value Kappa KA 0.1
Command: Purpose: Shortcut:	Lambda Set the lambda scaling parameter LA

Parameters: Example:	None to estimate lambda, or a number to fix it to value Lambda LA 0.8
Command: Purpose: Shortcut: Parameters: Example:	LoadModels To load models from a model file, see SaveModels LM A model file name LoadModels ModelFile.bin
Command: Purpose: Shortcut: Parameters: Example:	MLTries Set the number of times to find the maximum likelihood values, higher values are more consistent but take longer to run MLT Number of maximum likelihood tries, default 10. MLTries 35 MLT 100
Command: Purpose: Shortcut: Parameters: Example:	OU Set the lambda scaling parameter OU None to estimate OU, or a number to fix it to value OU OU 3.5
Command: Purpose: Shortcut: Parameters: Example:	Pis Set the base frequencies Pi Set the base frequencies estimates, est, emp, uni and none, emp for empirical frequencies, uni for uniform and none not to use any (all set to 1). Pis emp Pis uni Pis none
Command: Purpose: Shortcut: Parameters: Example:	Prior Set the prior for a parameter pr a parameter, a distribution type and parameters, distributions include, beta, gamma, uniform and exp Prior alpha1 exp 10 Prior q01 gamma 10 5 Prior q10 Beta 2.5 1 Prior q34 Uniform 0 1 Prior OU exp 1
Command: Purpose: Shortcut: Parameters: Example:	PriorAll Set the prior for all parameters PA A distribution type and parameters, see prior command PriorAll Exp 10 PriorAll Beta 1 7

Command: Purpose: Shortcut: Parameters: Example:	PriorCats Specify the number of categories to divide the prior into, default 100. PCat An integer > 1, PriorCats 200 PCat 50
Command: Purpose:	RateDev Sets the rate deviation parameter, effecting acceptance rate. The new version automatically finds a good acceptance rate. Setting it manually is an advanced option and should be avoided.
Shortcut: Parameters:	RD None to automatically estimate the parameter, an value to set all parameters to the same value, a parameter name and value to set a specific parameter to a value, only for continuous models.
Example:	RateDev RateDev 12.5 RateDev alpha-1 0.3
Command: Purpose: Shortcut: Parameters: Example:	Restrict Restrict a parameter or parameters to another parameter of a fixed value. Res A list of parameter to restrict, a parameter or fixed value to restrict to. Restrict alpha1 beta1 Restrict alpha1 beta1 alpha2 beta2 Restrict beta1 beta2 1.5
Command: Purpose: Shortcut: Parameters: Example:	RestrictAll Restrict all parameter to a parameter or fixed value ResAll A parameter or fixed value RestrictAll alpha1 ResAll 0.75
Command: Purpose: Shortcut: Parameters: Example:	RevJump Set a reverse jump analysis RJ A prior and prior parameter RevJump exp 10 RevJump Gamma 4 20 RJ Beta 5.0 2.5
Command: Purpose: Shortcut: Parameters: Example:	RevJumpHP Set a reverse jump analysis with a hyper prior RJHP A hyper prior RevJumpHP exp 0 100 RevJumpHP gamma 0 100 0 50
Command: Purpose: Shortcut:	Run Run the analysis RU

Parameters: Example:	None Run
Command: Purpose: Shortcut: Parameters: Example:	Sample Set the sample frequency SA An integer > 0 Sample 1000 Sample 250
Command: Purpose: Shortcut: Parameters: Example:	SaveModels Save the models to a file SM A file name to save the models to. SaveModels ModelFile.bin SM ModelFile.bin
Command: Purpose: Shortcut: Parameters: Example:	SaveTrees Save the sample of trees before analysis ST A filename to save the trees to SaveTrees STrees.trees
Command: Purpose: Shortcut: Parameters: Example:	Seed Set the random seed se An integer, > 0, to seed the random number from Seed 39362 Se 483
Command: Purpose: Shortcut: Parameters: Example:	Stones Initialise the stepping stone sampler st The number of stones to use and the number of iterations to use each stone for The alpha and beta parameters which specify the distribution the stone are drawn from can also be supplied. Stones 100 10000 Stone 100 25000 0.6 8.0
Command: Purpose: Shortcut: Parameters: Example:	Symmetrical Make restrictions to create a symmetrical matrix SYM none Symmetrical SYM
Command: Purpose: Shortcut: Parameters: Example:	TaxaInfo Show taxa names and numbers TI None TaxaInfo

Command:	TestCorrel
Purpose:	To set the correlation between traits to zero, used for model testing.
Shortcut:	TC
Parameters:	None
Example:	TestCorrel
Command:	UnRestrict
Purpose:	Remove a parameter restriction
Shortcut:	UNRes
Parameters:	A parameter to un restrict
Example:	UnRestrict q01
Command:	UnRestrictAll
Purpose:	Remove all restrictions
Shortcut:	UnResAll
Parameters:	none
Example:	none
Command:	VarRates
Purpose:	Use the variable rates model
Shortcut:	VR
Parameters:	None
Example:	VarRates

References

Felsenstein, J. (1973). "Maximum-likelihood estimation of evolutionary trees from continuous characters." <u>American journal of human genetics</u> **25**(5): 471.

Felsenstein, J. (2004). Inferring phylogenies, Sinauer Associates Sunderland.

Freckleton, R. P. (2012). "Fast likelihood calculations for comparative analyses." <u>Methods in Ecology</u> and Evolution **3**(5): 940-947.

Gilks, W. R., et al. (1996). Introducing markov chain monte carlo. <u>Markov chain Monte Carlo in</u> <u>practice</u>, Springer.

Green, P. J. (1995). "Reversible jump Markov chain Monte Carlo computation and Bayesian model determination." <u>Biometrika</u> **82**(4): 711-732.

Hansen, T. F. (1997). "Stabilizing selection and the comparative analysis of adaptation." <u>Evolution</u>: 1341-1351.

Kass, R. E. and A. E. Raftery (1995). "Bayes factors." Journal of the american statistical association **90**(430): 773-795.

Newton, M. A. and A. E. Raftery (1994). "Approximate Bayesian inference with the weighted likelihood bootstrap." Journal of the Royal Statistical Society. Series B (Methodological): 3-48.

Organ, C. L., et al. (2007). "Origin of avian genome size and structure in non-avian dinosaurs." <u>Nature</u> **446**(7132): 180-184.

Pagel, M. (1994). "Detecting correlated evolution on phylogenies: a general method for the comparative analysis of discrete characters." <u>Proceedings of the Royal Society of London. Series B:</u> <u>Biological Sciences</u> **255**(1342): 37-45.

Pagel, M. (1997). "Inferring evolutionary processes from phylogenies." <u>Zoologica Scripta</u> **26**(4): 331-348.

Pagel, M. (1999). "Inferring the historical patterns of biological evolution." <u>Nature</u> **401**(6756): 877-884.

Pagel, M. and A. Meade (2006). "Bayesian analysis of correlated evolution of discrete characters by reversible-jump Markov chain Monte Carlo." <u>The American Naturalist</u> **167**(6): 808-825.

Pagel, M., et al. (2004). "Bayesian estimation of ancestral character states on phylogenies." <u>Systematic biology</u> **53**(5): 673-684.

Tuffley, C. and M. Steel (1998). "Modeling the covarion hypothesis of nucleotide substitution." <u>Mathematical biosciences</u> **147**(1): 63-91.

Venditti, C., et al. (2011). "Multiple routes to mammalian diversity." <u>Nature</u> **479**(7373): 393-396.

Xie, W., et al. (2011). "Improving marginal likelihood estimation for Bayesian phylogenetic model selection." <u>Systematic biology</u> **60**(2): 150-160.